Warm up Solutions

$$f(x) = x^4 - 7x^2 + 6x$$

$$f(x) = x(x^3 - 7x + 6)$$

$$= x(x^3 + 0x^2 - 7x + 6)$$

If Factorable & integer roots then roots come from the following list

$$\{\pm 1, \pm 2 \pm 3 \pm 6\}$$

from $x^3 - 0x^2 - 7x + 6$

$$\begin{array}{c|cccc} 1 & 1 & 0 & -7 & 6 \\ & \downarrow & 1 & 1 & -6 \\ \hline & 1 & 1 & -6 & \boxed{0} \end{array}$$

So we know $x^3 - 0x^2 - 7x + 6 =$

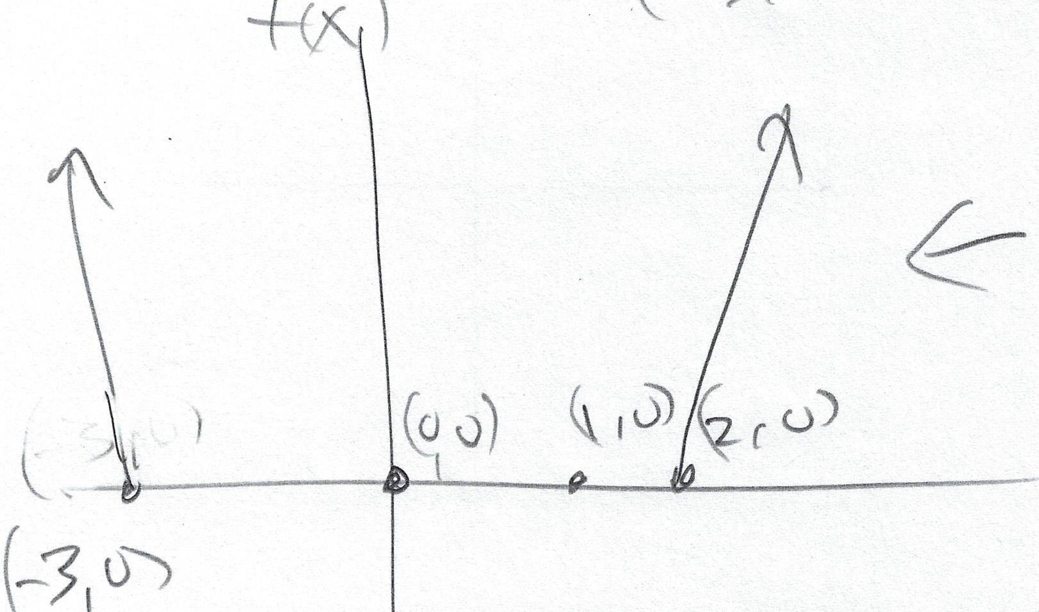
$$(x-1)(x^2 + 1x - 6)$$

Now $x^2 + 1x - 6$ is facto-able

$$x^2 + 1x - 6 = (x+3)(x-2)$$

So $f(x) = (x)(x-1)(x-2)(x+3)$

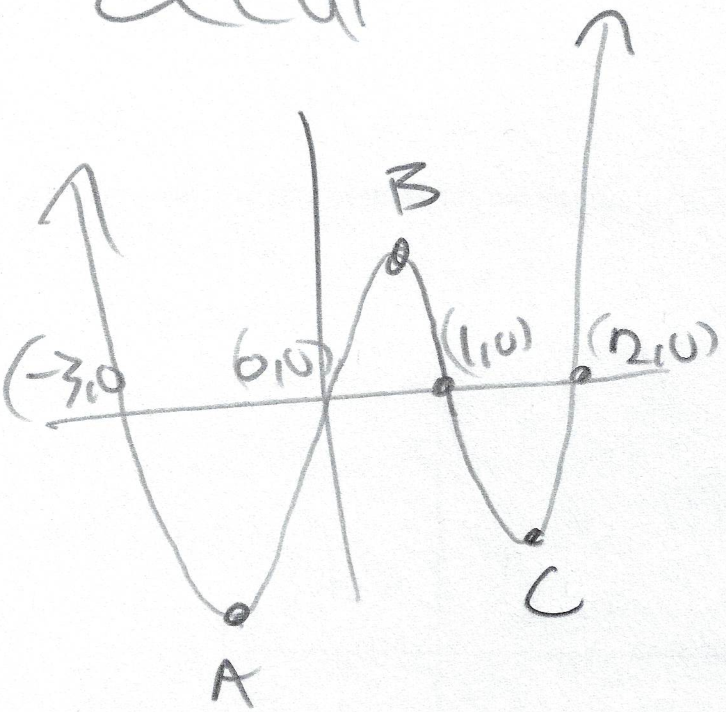
Roots are $(0,0)$ → also y intercept
$(1,0)$
$(2,0)$
$(-3,0)$



← we know this now because lead coefficient is positive an even degree

$$\left.\begin{array}{l} \text{as } x \to -\infty \ f(x) \to \infty \\ \text{as } x \to \infty \ f(x) \to \infty \end{array}\right\} \text{end Behavior}$$

without calculus or
a graphing calculator
we only know three
local extremes could
occur



$A \in (-3, 0)$
$B \in (0, 1)$
$C \in (1, 2)$

A or C could be global
extremes but B must be
a local max only

We can decribe behavio w/o knowing A, B, C values

for $x \in (-\infty, A)$ f(x) decreases

$x \in (A, B)$ f(x) increases

$x \in (B, C)$ f(x) decreases

$x \in (C, \infty)$ f(x) increases

$f(x) > 0$ for all $x \in (-\infty, -3) \cup (0, 1) \cup (2, \infty)$

$f(x) < 0$ for all $x \in (-3, 0) \cup (1, 2)$

$f(x) = x^4 - 7x^2 + 6x$

$f'(x) = 4x^3 - 14x + 6$

local ext-emes occur whe-

$f'(x) = 0$ but $f'(x) = 0$

is tough to find without

IVT or g-aphi-g calculato-

IVT intermediate Value Theorem

basically states over a closed

inte-val if $f(a) > 0$ then it

also becomes $f(b) < 0$ fo-

a continuous function

Then for some $c \in (a, b)$

$f(c) = 0$

CALCULUS SPOKEN HERE

$$f(x) = x^4 - 7x^2 + 6x$$

$$\begin{array}{r|rrrrr}
-3 & 1 & 0 & -7 & 6 & 0 \\
 & \downarrow & -3 & 9 & -6 & 0 \\
\hline
 & 1 & -3 & 2 & 0 & \boxed{0}
\end{array}$$

$$\boxed{\begin{aligned} f(x) &= (x+3)(x^3-3x^2+2x) \\ &= x(x+3)(x^2-3x+2) \\ &= x(x+3)(x-1)(x-2) \end{aligned}}$$

$$\begin{array}{r|rrrrr}
2 & 1 & 0 & -7 & 6 & 0 \\
 & \downarrow & 2 & 4 & -6 & \\
\hline
 & 1 & 2 & -3 & 0 & \boxed{0}
\end{array}$$

$$\boxed{\begin{aligned} f(x) &= (x-2)(x^3+2x-3) \\ &= x(x-2)(x^2+2x-3) = x(x-2)(x+3)(x-1) \end{aligned}}$$

$$f(x) = x^4 - 7x^2 + 6x$$

1	1	0	-7	6	0
	↓	1	1	-6	0
	1	1	-6	0	0

$$\boxed{\begin{aligned} f(x) &= (x-1)(x^3 + 1x^2 - 6x) \\ &= x(x-1)(x^2 + 1x - 6) \\ &= x(x-1)(x+3)(x-2) \end{aligned}}$$